Name: Solutions

This assignment consists of seven questions, each worth five points for a total of 35 points. To receive full credit you must **show all necessary work**. You should write your answers in the spaces provided, but if you require more space please *staple any extra sheets* you use to this assignment. If you are having trouble with any of the problems, look at the lecture notes and exercises in the lecture notes for help. Remember to start this assignment early, your next quiz is based on this assignment.

1. For each of the following, find a possible formula for the function represented by the data.

(a)

x	0	5	10	15
$f(x)$	11	14	17	20

+5 +5 +5

initial

+3 +3 +3

$y = mx + c$
$= \frac{3}{5}x + 11$

$m = \frac{14-11}{5-0} = \frac{3}{5}$

Answer: $f(x) = \frac{3}{5}x + 11$

(b)

t	0	1	2	3
$p(t)$	3.2	4.32	5.832	7.8732

+1 +1 +1

initial

+1.12 +1.512 ⇒ not linear

$\frac{4.32}{3.2} = 1.35$

$\frac{5.832}{4.32} = 1.35 \Rightarrow a = 1.35$

$\frac{7.8732}{5.832} = 1.35$

$p(t) = P_0 a^t$
$= 3.2(1.35)^t$

Answer: $p(t) = 3.2(1.35)^t$

(c)

s	1	2	3	4
$q(s)$	2.4	1.92	1.536	1.2288

+1 +1 +1

−0.48 −0.384 ⇒ not linear

$a = \frac{1.92}{2.4} = \frac{1.536}{1.92} = \frac{1.2288}{1.536} = 0.8$

$q(s) = P_0(0.8)^s$
$q(1) = P_0(0.8) = 2.4$

$\Rightarrow P_0 = \frac{2.4}{0.8} = 3$

Answer: $q(s) = 3(0.8)^s$

2. (a) Let $9^t = 13$. Solve for t using natural logarithms.

$\Rightarrow \ln(9^t) = \ln(13)$

$\Rightarrow t \cdot \ln(9) = \ln(13)$

Answer: $t = \frac{\ln(13)}{\ln(9)} \approx 1.167$ (3 d.p.)

(b) Let $12 = 7e^{-0.9t}$. Solve for t using natural logarithms.

$\Rightarrow \frac{12}{7} = e^{-0.9t}$

$\Rightarrow \ln(12/7) = \ln(e^{-0.9t})$

$\Rightarrow \ln(12/7) = -0.9t$

Answer: $t = \frac{\ln(12/7)}{-0.9} \approx -0.599$ (3 d.p.)

(c) Let $6e^{10t} = 11e^{8t}$. Solve for t using natural logarithms.

$\Rightarrow e^{10t} = \frac{11}{6} e^{8t}$

$\Rightarrow e^{2t} = \frac{11}{6}$

$\Rightarrow \ln(e^{2t}) = \ln(11/6)$

$\Rightarrow 2t = \ln(11/6)$

Answer: $t = \frac{\ln(11/6)}{2} \approx 0.303$ (3 d.p.)

3. A town has a population of 5,982 people at time $t = 0$. The population increases by 7.2% each year.

(a) Write a formula for the population, P, of the town as a function of years t.

$7.2\% = 0.072$

$P_0 = 5{,}982$

Answer: $P(t) = 5{,}982(1.072)^t$

(b) How many years will it take for the population to reach 12,000 people.

$P(t) = 12{,}000$

$\Rightarrow 5{,}982(1.072)^t = 12{,}000$

$\Rightarrow 1.072^t = \frac{12{,}000}{5{,}982}$

$\Rightarrow \ln(1.072^t) = \ln\left(\frac{12{,}000}{5{,}982}\right)$

$\Rightarrow t\ln(1.072) = \ln\left(\frac{12{,}000}{5{,}982}\right)$

Answer: $t = \frac{\ln\left(\frac{12{,}000}{5{,}982}\right)}{\ln(1.072)} \approx 10.013$ years (3 d.p.)

Cont.

4. If you need $42,000 in your bank in 12 years, to the nearest dollar, how much must you deposit now if the interest rate is 3.2%, compounded monthly. Give both the amount and the formula for the bank balance you used.

$A(12)$ $n=12$ t r

$$A(t) = P_0\left(1+\frac{r}{n}\right)^{nt}$$

$$\Rightarrow A(12) = P_0\left(1+\frac{0.032}{12}\right)^{12\times 12}$$

$$\Rightarrow 42{,}000 = P_0\left(1+\frac{0.032}{12}\right)^{144}$$

$$\Rightarrow P_0 = \frac{42{,}000}{\left(1+\frac{0.032}{12}\right)^{144}} \approx 28{,}622.14$$

Answer: $P_0 = 28{,}622.14$

Answer: $A(t) = 28{,}622.14\left(1+\frac{0.032}{12}\right)^{12t}$

5. In 1923, koalas were introduced on Kangaroo Island off the coast of Australia. In 1996 the population was 5,000. By 2005, the population had grown to 27,000, prompting a debate on how to control their growth and avoid koalas dying of starvation.

P_0 $P(9)$

(a) Assuming exponential growth, find the (continuous) rate of growth of the koala population between 1996 and 2005. Round your answer to 3 decimal places.

$$P(t) = P_0 e^{kt}$$ cont. rate of growth

$$\frac{P(9)}{P(0)} = \frac{P_0 e^{k\times 9}}{P_0} = e^{k\times 9} = \frac{27{,}000}{5{,}000}$$

$$\Rightarrow 9k = \ln\left(\frac{27{,}000}{5{,}000}\right)$$

Answer: $k = \dfrac{\ln\left(\frac{27{,}000}{5{,}000}\right)}{9} \approx 0.187$ (3.d.p)

(b) Find a formula, $P(t)$, for the population as a function of the number of years, t, since 1996.

Answer: $P(t) = 5{,}000e^{0.187t}$

(c) Estimate the population in the year 2021.

$$P(25) = 5{,}000e^{0.187(25)}$$

Answer: $\approx 536{,}163$

Cont.

6. Carbon-14 has a half life of 5,730 years. In 2018, a painting contains 91.3% of its carbon-14. Find a formula, $C(t)$, that gives the percentage of carbon-14 expected to be present in the painting t years after it was painted. Use this formula to estimate the year that this painting was painted.

0.913

$C(0) = P_0 = 100\% = 1$

$C(5{,}730) = e^{k \times 5730} = 50\% = 0.5$

$\Rightarrow \ln(e^{k \times 5730}) = \ln(0.5)$

$\Rightarrow k \times 5730 \ln(e) = \ln(0.5)$

$\Rightarrow k \times 5730 = \ln(0.5)$

$\Rightarrow k = \dfrac{\ln(0.5)}{5730}$

$\Rightarrow C(t) = e^{\frac{\ln(0.5)}{5730}t}$

$e^{\frac{\ln(0.5)}{5730}t} = 0.913$

$\Rightarrow \dfrac{\ln(0.5)}{5730}t = \ln(0.913)$

$\Rightarrow t = \dfrac{5730\ln(0.913)}{\ln(0.5)} \approx 752.425$

$2018 - 752.425 = 1265.575$

Answer: 1265

7. (a) Let $f(x) = 5x - 1$ and $g(x) = 3x + 2$. Find $g(f(x))$. Leave your answer in slope-intercept form.

$g(f(x)) = 3f(x) + 2 = 3(5x-1) + 2 = 15x - 3 + 2$

Answer: $g(f(x)) = 15x - 1$

(b) Let $f(x) = x - 2$ and $g(x) = x^2 + 8$. Find $f(g(7))$.

$g(7) = 7^2 + 8 = 49 + 8 = 57$

$f(g(7)) = f(57) = 57 - 2$

Answer: $f(g(7)) = 55$

(c) Let $f(x) = \sqrt{x+4}$ and $g(x) = x^2$. Find $f(g(-5))$.

$g(-5) = (-5)^2 = 25$

$f(g(-5)) = f(25) = \sqrt{25+4}$

Answer: $f(g(-5)) = \sqrt{29}$

The End.